NOTE ON ELECTRICAL RESPONSE IN A PIEZO-ELECTRIC PLATE TRANSDUCER WITH A PRESCRIBED INPUT

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ABSTRACT. The electrical signal produced by a transient mechanical force has been calculated for a piezoelectric plate with resistive loading at the electrical terminals.

INTRODUCTION

The investigation of responses (electrical or mechanical) in a piezoelectric transducer is doubtless, an important electromechanical problem in acoustics and ultrasonics, specially in the detection of ultrasonic waves. The mathematical treatment of the phenomena has been undertaken by Fillipczynski (1956), Redwood (1951, 1961, 1962). These papers have, in fact, contributed in a large measure to the recent studies in the topic by Sinha (1962a, 1962b, 1963, 1965) Giri (1965). As a sequal to this set of problems, the present note sets out to consider the problem of determining the electrical response in a piezoelectric plate transducer subjected to a mechanical force-input which is partly constant and partly transient.

PROBLEM, FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

We consider here a piezoelectric plate transducer executing vibration in the thickness mode. Let the thickness direction of the transducer be taken in the direction of the x-axis and let its extremities be $x = 0$ and $x = X$. Our problem consists in determining the electrical response in a transducer owing to some prescribed mechanical inputs, and certain mode of displacements.

It has been shown by Redwood (1962) that under certain assumptions, ζ, the mechanical displacement in the x-direction satisfies within or without the transducer an equation of the type

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{c}{\rho} \frac{\partial^2 \xi}{\partial x^2}$$

where c is the material constant and ρ is the density of the material of the transducer.

Let us seek a solution of the above equation in the form given by

$$\zeta = \phi \exp(-\omega t), \quad \omega > 0. \qquad \dots \quad (1)$$

so that ϕ satisfies

$$\frac{d^2\phi}{dx^2} - \frac{\omega}{v^2}\cdot\phi = 0 \qquad \dots \quad (2)$$

where

$$v^2 = \frac{c}{\rho}.$$

Solving equation (2) we get,

$$\phi = A \exp\left(-\frac{\omega x}{v}\right) + B \exp\left(\frac{\omega x}{v}\right) \qquad \dots \quad (3)$$

where A, B are constants.

Following Redwood (1961) the mechanical force F exerted on an area normal to x and the electrical voltage V across the transducer are calculated as follows :

$$F + hQ = \omega Z_c \exp(-\omega t)\left\{-A \exp\left(-\frac{\omega x}{v}\right) + B \exp\left(\frac{\omega x}{v}\right)\right\} \qquad \dots \quad (4)$$

$$V = -h\{(\xi)_X - (\xi)_0\} + \frac{Q}{C_0} \qquad \dots \quad (5)$$

where h is a piezoelectric constant of the material, Q the total charge at the surface of the transducer, C_0 the static capacitance of the transducer and Z_c the characteristic impedance of the material. Terms such as $(\xi)_X$ signify the value of ξ at $x = X$.

To ascertain the constants A and B, we must enumerate the boundary conditions of the problem. The most general type of this problem may be thought of by having a transducer of impedance Z_c situated between two systems of mechanical impedances Z_1 and Z_2. Then the boundary conditions are that the stresses and displacements are continuous at $x = 0$ and $x = X$. We write, at $x = 0$

$$(F_1)_0 = (F)_0$$

$$(\xi_1)_0 = (\xi)_0 \qquad \dots \quad (6)$$

and at

$$x = X$$

$$(F_2)_X = (F)_X$$

$$(\xi_2)_X = (\xi)_X$$

where ξ, F and V are given by the equations (1), (3), (4) and (5); and so

$$\xi = \exp(-\omega t)\left\{ A \exp\left(-\frac{\omega x}{v}\right) + B \exp\left(\frac{\omega x}{v}\right)\right\} \quad \dots \quad (7)$$

$$\xi_1 = \exp(-\omega t)\left\{ A_1 \exp\left(-\frac{\omega x}{v}\right) + B_1 \exp\left(\frac{\omega x}{v}\right)\right\} \quad \dots \quad (8)$$

$$F_1 = \omega Z_1 \exp(-\omega t)\left\{ -A_1 \exp\left(-\frac{\omega x}{v_1}\right) + B_1 \exp\left(\frac{\omega x}{v_1}\right)\right\} \quad \dots \quad (9)$$

$$\xi_2 = \exp(-\omega t)\left\{ A_2 \exp\left(-\frac{\omega x}{v_2}\right) + B_2 \exp\left(\frac{\omega x}{v_2}\right)\right\} \quad \dots \quad (10)$$

$$F_2 = \omega Z_2 \exp(-\omega t)\left\{ -A_2 \exp\left(-\frac{\omega x}{v_2}\right) + B_2 \exp\left(\frac{\omega x}{v_2}\right)\right\} \quad \dots \quad (11)$$

where the suffixes 1 and 2 denote the entities and constants of the systems of impedance Z_1 and Z_2, respectively.

METHOD OF SOLUTION

We assume that the transducer is connected to a high-input impedance of resistance R, so that when $Z_2 \to \infty$, $(\zeta)_x = 0$ and $A_2 = B_2 = 0$. From the conditions (6) the equations determining V are

$$V = -QR$$

$$A \exp\left(-\frac{\omega x}{v}\right) + B \exp\left\{\frac{\omega x}{v}\right\} = 0$$

$$A + B = A_1 + B_1 \quad \dots \quad (12)$$

$$\omega Z_1 \exp(-\omega t).(-A_1 + B_1) = \omega Z_c \exp(-\omega t).((-A+B) - hQ.$$

$$V = h(A+B)\exp(-\omega t) - \frac{V}{C_0 R}$$

Eliminating A, B, B_1 and Q we get,

$$V\left(1 + \frac{1}{C_0 R}\right) = \frac{h \exp(-\omega t)\left\{\exp\left(\frac{\omega X}{v}\right) - \exp\left(-\frac{\omega X}{v}\right)\right\}\left\{2A_1 Z_1 - \frac{hQ}{\omega}\exp(\omega t)\right\}}{(Z_c - Z_1)\exp\left(-\frac{\omega X}{v}\right) + (Z_c + Z_1)\exp\left(\frac{\omega X}{v}\right)} \quad \dots \quad (13)$$

The constant A_1 is to be determined from the applied mechanical input which is $F = F_0\{1-\exp(-kt)\}$; $K > 0$ and F_0 is a constant. Since this is introduced at $x = 0$, we have,

$$-A_1 \omega Z_1 \exp(-\omega t) = F_0\{1-\exp(-kt)\}.$$

so that

$$A_1 = -\frac{F_0\{1-\exp(-kt)\}}{\omega z_1} \exp(\omega t);$$

and equation (13) takes the form.

$$V = \frac{hc_0 R}{\omega(1+c_0 R)} \left\{ \exp\left(-\frac{\omega X}{v}\right) - \exp\left(\frac{\omega X}{v}\right) \right\}$$

$$\frac{2F_0\{1-\exp(-kt)+hQ\}}{\left\{(Z-Z_e)\exp\left(-\frac{\omega X}{v}\right) + (Z_c+Z_1)\exp\left(\frac{\omega X}{v}\right)\right\}}$$

This gives the electrical voltage i.e. the electrical response which is obviously a constant in part and transient in part.

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